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Enhancing Grade 4 students' mathematical problem-solving skills using manipulatives

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ABSTRACT

Manipulatives enhance learners' mathematical problem-solving skills by providing concrete tools to visualize and approach problems. This study investigated their effectiveness among Grade 4 learners at San Rafael Integrated School using a quasi-experimental design. Data were collected through pre-tests, post-tests, and a validated researcher-created questionnaire. The tests measured problem-solving performance before and after the intervention, while the questionnaire explored learners' experiences with manipulatives. The study aimed to compare the initial problem-solving abilities of the experimental and control groups, analyze differences after the intervention, and evaluate learners' feedback on using manipulatives. Pre-test results showed no significant difference between the groups, indicating similar starting points. Post-test results, however, demonstrated that the experimental group using manipulatives significantly outperformed the control group, highlighting the effectiveness of this approach. The findings reveal the limitations of traditional teaching methods and underscore the value of manipulatives in improving mathematical problem-solving skills. Integrating manipulatives into regular instruction is recommended to better support learners in mastering essential mathematics concepts.

Keywords: Experimental learning, manipulatives, problem-solving skills, San Rafael Integrated School, teaching methods

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INTRODUCTION

Integrating manipulatives in enhancing students' problem-solving skills involves using physical or virtual objects that students can manipulate to better understand and solve mathematical problems (Bartolini and Martignone, 2020). These manipulatives serve as a bridge between abstract concepts and concrete understanding, making it easier for students to grasp complex ideas (Aghli et al., 2016). According to a research by Lanante (2019), which involved Grade 2 elementary students from the Central Philippines, the usage of manipulative materials improved students' problem-solving abilities and enrichment activities.

When students engage with manipulatives, they can explore mathematical concepts hands-on, which fosters deeper comprehension and retention of the material (Larbi and Mavis, 2023). This hands-on approach not only aids in understanding but also enhances students' critical thinking and problem-solving abilities by allowing them to experiment with different strategies in a risk-free environment (Carbonneau et al., 2020). Moreover, manipulatives can be particularly beneficial for younger students or those with different learning needs, as they provide a tangible way to interact with mathematical concepts, thereby improving engagement and motivation (Pires et al., 2019).

A study from the Philippines examines the impact of using manipulatives like fraction bars and Cuisenaire rods in teaching fractions to elementary school students, the researcher emphasizes that manipulatives helped students visualize the relationship between fractional parts, thus facilitating a deeper understanding of abstract ideas (Tiongco, 2018). Another study that conducted in Odiongan Central School (OCS), Division of Gingog City, Misamis Oriental, and its respondents were the two sections of Kinder-Daffodil. They stated that math manipulatives are fun and using these concrete tools makes the lesson easy to understand (Guanzon-Pisaras, 2020). Khalid et al. (2020) aimed to improve creativity in mathematics education by teaching problem-solving techniques, which showed significant increases in creativity and problem-solving scores. Chiang Mai Rajabhat University Demonstration School in Thailand reveals that third-grade students' critical thinking skills in mathematics need improvement, particularly in reading analog clocks (Dewi et al., 2020). While Singaporean children excel in international mathematics tests, possibly due to better textbooks offering more reasoning and fewer strategy steps, this indicates better scaffolding for word problem solving, as asserted by Ebikawa (2023).

Furthermore, in line with Astrani et al. (2017), problem-solving ability is the ability to understand the purpose of a problem and the rules that can be applied to solve it. This is the heart of creative mathematics (Yazgan, 2015), ignites imagination (Bruce, 2015), sparks creativity (Suastika, 2017), and unlocks deeper understanding (Mitchell and Walinga, 2017). A study from Central Luzon, Philippines, states that improving students' problem-solving abilities remains challenging for teachers and students. The results showed that cognitive domains in knowledge, comprehension, application, and evaluation were significant determinants of problem-solving ability (Jimenez, 2020). Bungaw-Abarquez (2020) state that, the use of manipulative in teaching mathematics is more effective than the use of conventional method. This study was conducted in two grade 3 sections of San Agustin Elementary School in the school year 2016-2017.

A study from US, that focuses on primary and middle school students basically from Kindergarten to eight, authors argue that manipulatives are critical tools for bridging the gap between abstract mathematical concepts and students' understanding, especially for those who struggle with more abstract topics (Moyer and Milewicz, 2020). As Bartolini and Martignone (2020) define them, these tangible objects, whether concrete-like blocks or virtual-like software, allow students to explore and internalize mathematical ideas through hands-on interaction. Thus, manipulatives are considered helpful to students in learning mathematics and a tool teachers use to introduce mathematical concepts and assess their understanding, according to Larbi and Mavis (2023). A study of (Aghli et al., 2016) was conducted from India with the respondents from grade four students that, through solving mathematical problems. A study also from China that use manipulatives (such as algebra tiles and base-ten blocks) in primary schools with the respondents from grade one to six, showed that manipulatives significantly improved students' understanding of abstract and mathematical concepts (Wang and Li, 2021).

The benefits extend beyond concrete materials. According to the study by Carbonneau et al. (2020), they contribute by investigating the mutual impact of perceptually rich manipulatives and collaboration on students' mathematical problem-solving and perseverance, highlighting the interconnected nature of these elements in shaping effective learning environments. Another study by Dewi and Verawati (2021) examined the effect of manipulative games on enhancing fundamental motor skills in elementary school students, providing valuable insights into the diverse applications of manipulative activities in educational settings. Moreover, interaction with objects may support the development of different strategies by diminishing cognitive load and freeing up working memory, given that the perceived entities are cognitively available through the objects that represent them in space (Manches and O'Malley, 2016). Physical manipulatives, such as virtual and tactile items, positively impacted mathematical ability, improved opportunities for action, and fostered a more thorough comprehension of fundamental mathematical ideas (Pires et al., 2019).

The researcher aimed to explore the effectiveness of manipulatives as a strategy to enhance problem-solving skills in Grade 4 pupils. By examining the impact of manipulatives on pupils' comprehension, fluency, and problem-solving abilities, the researcher seeks to contribute to the existing knowledge on effective interventions for boosting mathematical skills in this age group. Furthermore, the study sought to provide insights into the effectiveness of manipulatives as a pedagogical tool, inform future instructional practices, and contribute to a richer educational experience for Grade 4 students at San Rafael Integrated School.

MATERIALS AND METHODS

Research locale

The study was conducted at San Rafael Integrated School, located in Proper, San Rafael, Cateel, Davao Oriental. This school, part of Cateel District 2, is one of the two integrated schools in the area, along with Taytayan Integrated School. The researchers chose this school as the site for their study based on observations during their teaching practicum in FS 100 and their teaching assistantship in FS 101. During these periods, they identified significant challenges among students in developing effective problem-solving skills, particularly in mathematical contexts.

Research instrument

The researcher utilized an achievement test focusing on solving routine and non-routine problems in real-life situations involving the perimeter of squares, rectangles, triangles, parallelograms, and trapezoids. This test allowed students to analyze problems, think critically, and apply their problem-solving skills effectively. This distinct tool was developed to measure learning in respondents through initial and final evaluations, with its effectiveness confirmed through detailed validation and reliability tests.

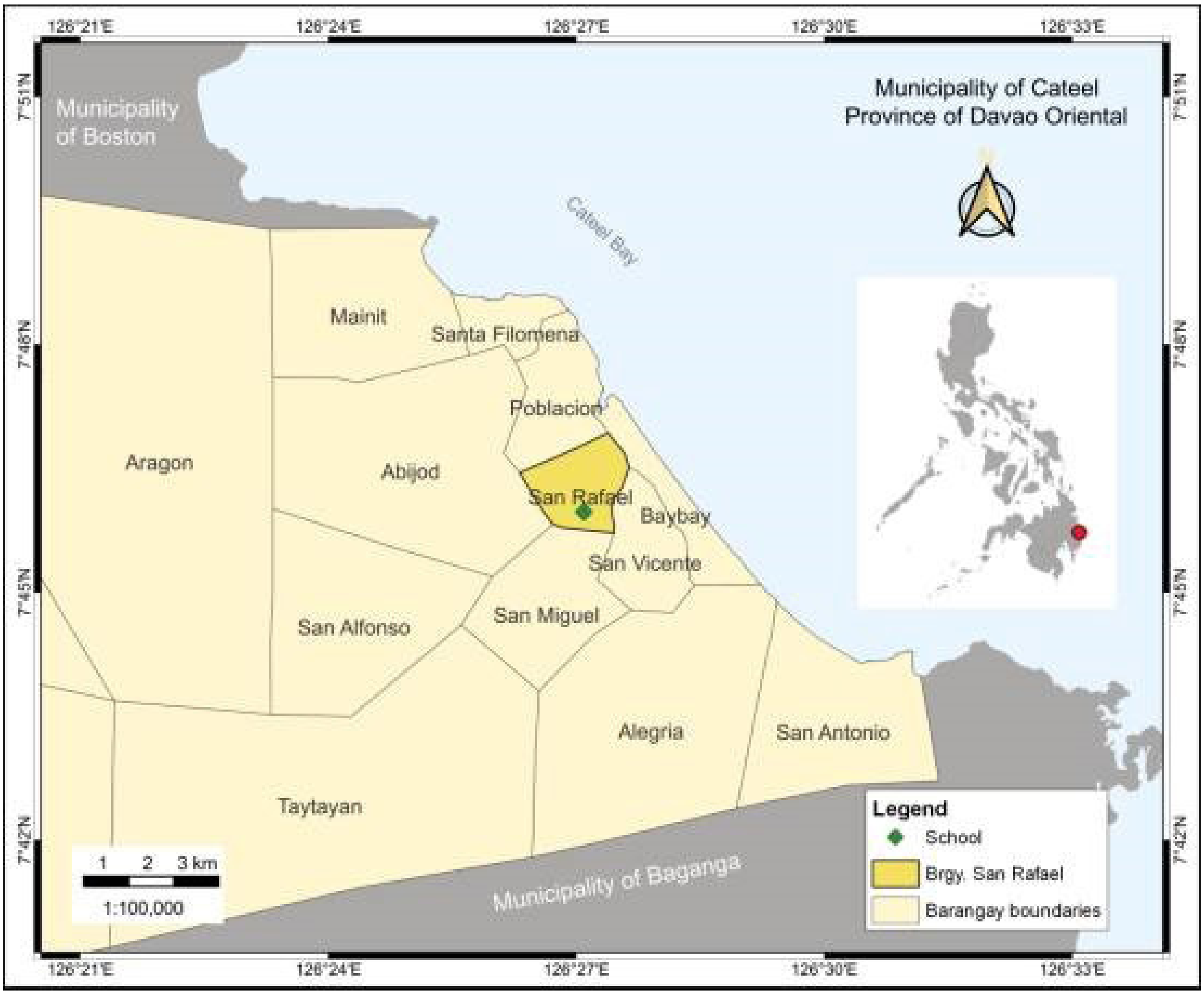

Figure 1. Map of Cateel, Davao Oriental, and San Rafael Integrated School.

Data collection

Prior to data collection, the researchers obtained ethical clearance from the Research Ethics Office (REO) at Davao Oriental State University—Cateel Extension Campus. They developed a questionnaire designed to assess problem-solving skills, which underwent validity and reliability testing to ensure its appropriateness for the study. After securing approval from the principal of San Rafael Integrated School, the researchers submitted a formal request to the class advisers to conduct the study and administer the tool to the students in the experimental group.

To ensure a fair distribution of participants, the one-coin-toss sampling technique was employed. This method involved assigning each section a numerical value, flipping a coin for each section, and allocating them to either the experimental or control group based on the outcome of the coin toss (Figure 2B and C). Acknowledging that student absences are inevitable, the study included 30 students from each section, resulting in a total of 60 respondents.

The respondents were first given a pre-test, accompanied by a consent form that requested their permission to participate in the research process. The consent form also allowed respondents to optionally provide their names and required their signatures. Once the pre-test was completed, the questionnaires were collected, and the data were totaled, tallied, encoded, analyzed, and interpreted.

During the instructional phase,s the control group was taught problem-solving skills using traditional methods, while the experimental group received instruction using manipulatives designed to enhance their problem-solving abilities in various mathematical contexts. These manipulatives provided hands-on experiences that helped students better understand and apply problem-solving strategies. Following these instructional sessions, both groups were given post-tests to evaluate the effectiveness of the teaching methods and to determine whether the use of manipulatives had a positive impact on the students' ability to solve routine and non-routine problems.

Figure 2. Conduct of Pilot Testing of the test-questions at Taytayan Integrated School (A); Conduct of pre-test for the control group (B); Conduct of pre-test for the experimental group (C); Conduct of post-test for the experimental group (D); Conduct of post-test for the control group.

Finally, the post-test questionnaires were collected, and the data were totaled, tallied, encoded, analyzed, and interpreted to draw conclusions about the effectiveness of using manipulatives as an intervention to improve students' problem-solving skills.

Data analysis

Content Validity. The content validity of the instrument was confirmed through the rigorous application of Aiken's V coefficient. This method involves an expert review of how well the tool matches educational goals, its necessity, and the quality of its content. The process yielded an impressive Aiken V coefficient of 0.94, highlighting its strong validity (Sireci and Bond, 2014).

Reliability. The instrument's reliability was evaluated using Cronbach's alpha, a statistical tool for measuring internal consistency. This assessment produced a Cronbach's alpha of 0.67, demonstrating a satisfactory reliability level for its purposes (Ahdika, 2017). These comprehensive validation and reliability checks emphasize the tool's capability to accurately and reliably assess participants' learning outcomes, proving its utility and reliability in educational research contexts.

The K-12 Department of Education grading system was employed to analyze and arrive at a trustworthy and accurate interpretation of the gathered data from the pre-test and post-test questionnaire responses from the experimental and control groups. Mean scores and an independent sample t-test were used to determine and analyze the findings. Mean. This statistical tool was used to determine (1) the average pre-test problem-solving achievement between the control group and the experimental group and (2) the average post-test problem-solving achievement between the control group and the experimental group. The results were interpreted using a grading scale with corresponding classifications.

Table 1. K to 12 grading scale and interpretation.

Grading scale	Interpretation
90-100	Outstanding
85-89	Very satisfactory
80-84	Satisfactory
75-79	Fairly satisfactory
75 Below	Did not meet expectations

T-test. This statistical test was used to determine (2) the significant difference in pre-test ressults between the controlled and experimental groups and (4) the significant difference in post-test scores between the controlled and experimental groups.

Table 2. Table of interpretation.

p-value	Interpretation
Less than 0.05	There is a significant difference.
0.05 or more	There is no significant difference.

Analysis of Covariance (ANCOVA)

To further strengthen the analysis and control for any pre-existing differences between the control and experimental groups, an Analysis of Covariance (ANCOVA) was employed. ANCOVA allows for a more accurate determination of the impact of the intervention (manipulatives) by adjusting the post-test scores based on the learners' pre-test performance.

Covariate:

The pre-test scores were used as the covariate to account for baseline differences in problem-solving skills between the two groups. By controlling for these initial differences, the ANCOVA provided a clearer analysis of the impact of manipulatives on learners' post-test results.

Dependent variable:

The post-test scores were treated as the dependent variable, representing the learners' problem-solving abilities after the intervention.

Independent variable:

The group assignment (experimental or control) was the independent variable, indicating whether the students were part of the group that used manipulatives or the one that received traditional instruction.

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Procedure:

1. Checking Assumptions: Assumptions of ANCOVA, including homogeneity of regression slopes and normal distribution of residuals, were verified to ensure the validity of the analysis.
2. Adjustment of post-test scores: The post-test scores were adjusted for pre-test differences to isolate the effect of the intervention (use of manipulatives).
3. Main analysis: After adjusting for pre-test scores, the ANCOVA compared the post-test scores of the experimental and control groups.

RESULTS

Pre-test problem-solving test score achievement

This study compared students' problem-solving skills in an experimental group (n = 30) with a control group (n=30) before introducing manipulatives and traditional problem-solving methods. Both groups underwent a pre-test to assess their baseline problem-solving skills, specifically their ability to solve routine and non-routine problems in real-life situations involving the perimeter of squares, rectangles, triangles, parallelograms, and trapezoids. The statistical analysis of the pre-test scores is detailed in Table 3. The results indicated that neither the control nor the experimental group met the expectations outlined in the K to 12 grading system.

The data indicates that both the control and experimental groups performed similarly on the pre-test, with mean scores of 10.27 and 10.63, respectively. According to the K to 12 grading scale, these scores correspond to grade percentages of 67.12% for the control group and 67.55% for the experimental group. Both percentages fall into the "Did Not Meet Expectations" category, which is below the minimum proficiency threshold of 75%. The standard deviations of 2.75 for the control group and 3.63 for the experimental group suggest variability in individual performance, but overall, the results demonstrate that both groups had comparable levels of problem-solving skills before the introduction of manipulatives in the experimental group.

Table 3. Level of pre-test scores between the control and experimental groups.

Group	Total score	SD	Mean	Grade percentage	Remarks
Control	30	2.75	10.27	67.12	Did not meet expectations
Experimental	30	3.63	10.63	67.55	Did not meet expectations

Difference in the pre-test scores

The significant difference between the control and experimental groups' mean pre-test scores was analyzed. The statistical analysis of these scores is summarized in Table 4. This indicated that the mean pre-test score for the control group was 10.27 with a standard deviation of 2.75, while the experimental group had a mean score of 10.63 with a standard deviation of 3.63. The t-value was 0.320, and the *p*-value was 0.750, indicating no significant difference between the two groups' pre-test scores. This lack of significant difference suggests that both groups started at a similar level of problem-solving proficiency before the introduction of manipulatives in the experimental group. The similar baseline performance indicates that any differences observed in the post-test results can be more confidently attributed to the intervention rather than pre-existing disparities in ability. The data also highlight that students faced challenges in problem-solving skills, as reflected by their low pre-test scores, regardless of the teaching method initially employed.

Table 4. Mean comparison between pre-test scores of control and experimental group.

Group	Mean	SD	*t*-value	*p*-value	Interpretation
Control Experimental	10.27 10.63	2.75 3.63	0.320	0.750	Pre-test scores between the two groups do not differ significantly.

Post-test problem-solving test score achievement

The analysis of post-test problem-solving scores for both the control and experimental groups is presented in Table 5. The control group achieved a mean post-test score of 12.53, with a standard deviation of 2.19, and a grade percentage of 70.88, which did not meet expectations. On the other hand, the experimental group obtained a mean score of 22.23, with a standard deviation of 3.06, resulting in a grade percentage of 87.05, which was classified as very satisfactory. This notable difference in post-test scores indicates that the experimental group, which utilized manipulatives, performed significantly better than the control group. These results suggest that manipulatives were effective in improving students' problem-solving skills, highlighting the value of alternative teaching strategies in enhancing learning outcomes, especially in challenging areas.

Table 5. Level of post-test scores between the control and experimental groups.

Group	Total score	SD	Mean	Grade percentage	Remarks
Control	30	2.19	12.53	70.88	Did not meet expectations
Experimental	30	3.06	22.23	87.05	Very satisfactory

Difference in the post-test scores

The significant difference in post-test scores between the control and experimental groups was analyzed, with the results summarized in Table 6. The analysis revealed a substantial difference in the mean post-test scores for the two groups. The control group had a mean score of 12.53 with a standard deviation of 2.75, while the experimental group had a mean score of 22.23 with a standard deviation of 3.06. The t-value was -14.12, and the *p*-value was 0.000, indicating a statistically significant difference between the two groups. This significant difference suggests that the experimental group, which used manipulatives, demonstrated a markedly improved performance in problem-solving skills compared to the control group, which did not use manipulatives. The results underscore the effectiveness of manipulatives as a teaching strategy for enhancing students' problem-solving skills.

Table 6. Mean comparison between post-test scores of control and experimental group.

Group	Mean	SD	*t*-value	*p*-value	Interpretation
Control Experimental	12.53 22.23	2.75 3.06	-14.12	0.000	Pre-test scores between the two groups do not differ significantly.

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DISCUSSIONS

The findings of this study highlight the crucial role of problem-solving in enhancing students' overall mathematical performance. Beyond being a core skill in mathematics, problem-solving is a vital life skill that supports effective reasoning, critical thinking, and decision-making in academic and real-world contexts.

The results demonstrated that students using manipulatives significantly improved their problem-solving abilities, validating the potential of experiential learning methods to enhance educational outcomes. These findings align with Abramovich and Freiman (2023), who reviewed problem-solving challenges in mathematics across diverse student populations. Previous studies (DNEA, 2021; Raoano, 2016; Naukushu and Chirimbana, 2012) have identified inadequate problem-solving skills as a global issue, often stemming from language barriers, conceptual misunderstandings, and ineffective teaching techniques.

Researchers such as Al-Mutawah (2019) emphasize the need for a strong foundation in mathematical concepts and language to improve problem-solving abilities. Similarly, Phonapichat et al. (2014) and Angateeah et al. (2017) found that difficulty in understanding key mathematical terms and concepts often leads to poor performance. Ball et al. (2016) further highlighted the role of language comprehension in tackling mathematical problems.

The pre-test comparison revealed no significant difference between the control and experimental groups, suggesting similar initial abilities. Factors such as limited exposure to critical thinking strategies and traditional teaching methods likely contributed to the students' baseline performance (Doorman et al., 2017). However, post-test results indicated a significant improvement in the experimental group, consistent with studies supporting the use of manipulatives to enhance problem-solving skills (Haber, 2017; Ross, 2018). Manipulatives facilitate visualization, helping students better understand and engage with mathematical material (Zimmermann and Cunningham, 2021).

The success of the experimental group aligns with Kolb's Experiential Learning Theory (Burke, 2020), which emphasizes the value of hands-on learning in fostering critical and creative thinking. Manipulatives provided students with practical experiences that deepened their understanding and improved their ability to solve complex problems (Pedersen et al., 2005; Presmeg, 2006).

In contrast, the control group, which relied on traditional instruction, showed no significant improvement. This finding underscores the limitations of conventional teaching methods in equipping students with effective problem-solving strategies. The marked difference in post-test scores between the two groups reinforces the need for innovative, student-centered teaching approaches that address diverse learning needs (Lesh, 1981; Cooper and Dunne, 1999).

CONCLUSION

Incorporating manipulatives into the curriculum provides students with an interactive and engaging learning experience, which is crucial for grasping abstract concepts and solving complex problems. This hands-on approach encourages active learning and supports students in developing a deeper understanding of mathematical concepts. The study highlights the necessity for educational systems to move towards more innovative and student-centered teaching methods that cater to diverse learning styles and needs.

Based on the study's findings, several key recommendations are proposed. First, educational institutions should make manipulatives a standard part of their mathematics curriculum to boost problem-solving abilities. Second, teachers should receive professional development training focused on effectively integrating manipulatives into their teaching practices. Finally, further research should explore the long-term benefits of using manipulatives and their applicability across various mathematical concepts and grade levels. By implementing these recommendations, educational systems can ensure that students are better prepared for academic challenges and real-world problem-solving scenarios.

In conclusion, this study affirms the importance of adopting innovative teaching strategies like manipulatives to enhance student learning outcomes. The significant improvement observed in the experimental group highlights the potential for such approaches to transform mathematics education. As educators and policymakers strive to improve educational standards, the findings of this study offer valuable insights into how teaching methodologies can be adapted to meet the evolving needs of students. By prioritizing effective, hands-on learning experiences, we can foster a generation of learners who are better equipped to succeed in mathematics and beyond.

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AUTHORS CONTRIBUTIONS

Conceptualization, N.K.D.L and S.R. O.G; methodology, N.K.D.L and S.R.O.G; software, N.K.D.L and S.R.O.G; validation, N.K.D.LS., R.O.G and B.L.S; formal analysis, N.K.D.LS., R.O.G and B.L.S; demonstration, N.K.D.L and S.R.O.G; resources, data curation, N.K.D.L and S.R.O.G; writing-original draft preparation, N.K.D.L and S.R.O.G; writing-review editing, B.L.S; visualization, B.L.S; supervision B.L.S; project administration, N. K.D.LS., R.O.G and B.L.S; funding acquisition, N.K.D.L and S.R.O.G.

CONFLICT OF INTEREST

The authors have no conflict of interest in the result of the study.

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