DAVAO RESEARCH JOURNAL
https://davaoresearchjournal.ph P-ISSN 2244-4432 E-ISSN 2984-7125

Utilizing the lattice method to enhance multiplication automaticity in Grade 4 Pupils

Crezel Jane F. Moreno, Bryan L. Susada*

Program of Bachelor of Elemetary Education, Davao Oriental State University, Municipality of Cateel, Davao Oriental, 8205 Philippines, ORCID, Crezel Jane Moreno https://orcid.org/0009-0001-5741-0284, Bryan L. Susada https://orcid.org/0000-0003-1268-7973

**Corresponding author: brylsusada@gmail.com*

Submitted: 01 Aug 2024
Revised: 20 Aug 2024
Accepted: 06 Sep 2024
Published: 27 Sep 2024

OPEN ACCESS

ABSTRACT

This study introduced a new method to help students who struggle with multiplying multiple digits. This study examined the effect of the lattice approach on multiplication automaticity among Grade 4 students at Cateel Central Elementary School during the school year 2023-2024. Utilizing a quasi-experimental methodology, the study compared two intact groups: a control group with 22 respondents taught using traditional multiplication methods, and an experimental group with 21 respondents taught using the lattice method. Pre-test results indicated that neither the control nor the experimental group met the expectations set by the K to 12 grading system, with grade percentages of 65.68 and 67.22, respectively. Statistical analysis showed no significant difference between the pre-test scores of the two groups, suggesting a similar initial level of multiplication proficiency. Post-test results, however, revealed a significant improvement in the experimental group's performance, with a grade percentage of 84.77 compared to the control group's 74.70. The statistical analysis confirmed this difference as significant, with a t-value of -3.383 and a p-value of 0.002. The findings demonstrate the superior efficacy of the lattice method over traditional teaching methods in enhancing multiplication skills among Grade 4 students.

Keywords: Cateel, Lattice method, long method, math education, multiplication

How to cite: Moreno, C. J. F., and Susada, B. L. (2024). Utilizing the lattice method to enhance multiplication automaticity in Grade 4 Pupils. *Davao Research Journal*, 15(3), 122-132. https://doi.org/10.59120/drj.v15i3.258

INTRODUCTION

Many students need help to compute a multi-digit multiplication process. However, kids are only acquainted with using a specific technique to handle multiplication operations (Rebekawati, 2017). The Philippines was among the countries that produced the least number of math competency for young learners, according to the findings of the Programme for International Student Assessment (PISA) 2022 (Organisation for Economic Co-operation and Development, 2023; Balancio, 2023; Santos, 2023). They are made worse by conventional teaching strategies that put procedural fluency ahead of conceptual understanding (Lopez, 2020; Qetrani et al., 2021).

Mathematics is essential for understanding the world, developing mental discipline, and a fundamental component of human thought. In this field, multiplication is necessary, serving as a fulcrum for advancing mathematical understanding (Lewis, 2016; Gardiner, 2016; ICMI, 2023). Beyond simple math, multiplication is essential because it is the foundation for more advanced ideas and practical applications (Rizzo, 2023). Researchers (Boaler, 2015a; Newton et al., 2024) have pointed out that knowing multiplication's principles is crucial, as it goes beyond the traditional comprehension of facts committed to memory. Educational organizations like the National Council of Teachers of Mathematics (NCTM) support a holistic approach and real-world applications, highlighting the transforming power of multiplication instruction (Goos et al., 2020; Rineck, 2020).

However, difficulties still exist despite the significance of multiplication instruction (Van der Ven et al., 2015; Larsson, 2016; Baker and Cuevas, 2018; del Carmen Chamorro, 2021; Dotan and Zviran-Ginat, 2022). Thus, in mathematics, the percentage of students who scored below a baseline level of competency (Level 2) remained relatively the same from 2018 to 2023 (Organisation for Economic Co-operation and Development, 2023; Balancio, 2023; Santos, 2023)Scholars (Chevalier and Buckles, 2019) have highlighted the difficulties pupils have when learning multiplication, which highlights the need for creative solutions to improve learning results.

Teaching long multiplication presents a notable challenge due to the cognitive burden it places on students, as highlighted by research (Moussa-Inaty et al., 2020). This approach involves several steps, demanding sustained attention and concentration from students throughout (Arguel et al., 2017; Moussa-Inaty et al., 2020). The algorithm's intricacies may need to be clarified and more manageable for some learners, impeding their comprehension of the concept (Arguel et al., 2017; Richey et al., 2019).

According to research on mathematics education, conceptual and visual approaches —like the lattice method—improve learning outcomes and increase students' engagement and effectiveness with multiplication instruction (Özenç et al., 2020; Sarkingobir et al., 2023). The lattice approach, a possible remedy, is the subject of this study. Motivated by the need to tackle difficulties in teaching multiplication, teachers have resorted to cutting-edge techniques such as the lattice approach (Milton et al., 2019; Cardino and Ortega-Dela Cruz, 2020). The researcher selected this study based on its empirical nature, observed during assessment in FS 100, which revealed numerous students facing challenges in multiplying 2-3 digits. Despite being in an intermediate education, Grade 4 students at Cateel Central Elementary School continued to struggle with applying long multiplication methods or traditional ways of teaching the multiplication process. Thus, this study aimed to offer valuable insights into the efficacy of the lattice method in improving multiplication automaticity among Grade 4 pupils at Cateel Central Elementary School.

METHODS

Description of study area

The study was conducted at Cateel Central Elementary School on Castro Avenue, Poblacion, Cateel, Davao Oriental. This school is part of the Cateel 1 district and is adjacent to Cateel Vocational High School. It is the largest school in the municipality of Cateel in terms of both student enrollment and facilities, owing to its central location. As of 2024, the current principal is Mr. Constantino R. Bagumba, Principal II. The researcher selected this school based on observations made during an initial assessment in FS 100. This assessment highlighted that many Grade 4 students struggled with solving multiplication problems involving multi-digit numbers.

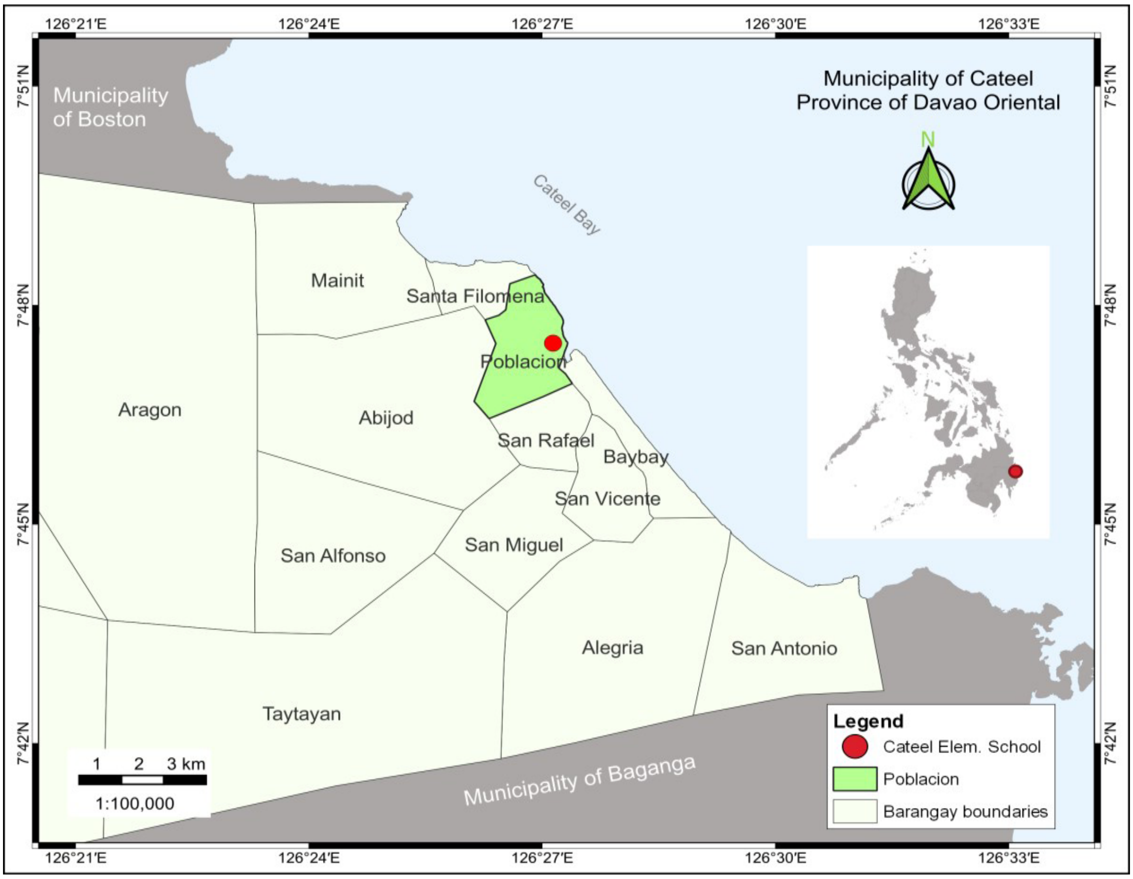

Figure 4. Map showing Cateel Central Elementary School (CCES).

Research design

In a quasi-experiment, other things, like existing groups or natural divisions, might decide who gets the new teaching method (Thomas, 2020). People use quasi-experimental designs when they cannot easily or ethically randomly assign groups (Thomas, 2020). A quasi-experimental design is appropriate for this study since it works with pre-existing groups (the control and experimental groups) that cannot be assigned randomly. Research Instrument

The researcher utilized an achievement test focusing on multiplying numbers up to 3-digit by 2-digit numbers. This test allowed students to recall multiplication concepts, analyze problems, and apply their learning effectively. This distinct tool was developed to measure learning in respondents through initial and final evaluations, with its effectiveness confirmed through detailed validation and reliability tests.

Data collection

In gathering data for this study, several key steps were undertaken to ensure the validity and reliability of the findings. First, the researcher sought permission to conduct a survey involving school heads and classroom advisers, laying

DOI: https://doi.org/10.59120/drj.v15i3.258

the groundwork for the study. A pilot test was then conducted which involved a sample size of 63 respondents from San Rafael Elementary School, which is a different institution from the primary study area (Figure 2A). In the pilot test, two sections were included to ensure a robust evaluation of the reliability and validity of the questionnaire. Since it was a one-take exam, all students present in those sections were included as respondents, and there was no need to exclude any students. It aimed solely to assess the questionnaire's reliability and validity, allowing for all present students to participate without exclusions. Any questions that were found to be invalid or unreliable were subsequently removed. It is important to note that the respondents from the pilot test were not included in the later analysis of the main study.

Next, the researcher consulted with the principal and grade 4 teachers to select two sections willing to participate in the study. A coin toss was used to determine which group would serve as the control group and which would be the experimental group (Figure 2B and C). The selection of the experimental and control groups was done using a coin flip, a method chosen for its simplicity and fairness in random assignment. The rationale behind this decision was to ensure that each group had an equal chance of being selected, thereby minimizing any potential bias. While other methods, such as drawing lots, could have been used, the coin flip was deemed sufficient for the purpose of this study. The experimental group with 21 respondents received instructionusing the lattice approach, while the control group with 22 respondents was taught using the conventional multiplication method. The reason why there were fewer respondents in the actual study compared to the pilot testing is that, to maintain the integrity and accuracy of the results, students who were absent for either the pre-test or post-test were excluded from the analysis. This was done to ensure that the data accurately reflected the impact of the intervention being studied, as incomplete data sets could skew the results and undermine the study's validity.

Figure 2. Conduct of pilot testing of the test-questions at San Rafael Integrated School (A); Conduct of pre-test for the control (B); Experimental group (C); Conduct of post-test for the experimental group (D).

The difference in the number of students between the experimental and control groups (21 in one group and 22 in the other) is because the groups were from different classrooms, and the initial allocation was based on the number of students present in each room. Additionally, some students were excluded from the final analysis because they did not complete the pre-test and post-test—some were absent for the pre-test and others for the post-test. Despite this discrepancy, the unequal sample sizes are not an issue for this study, as the independent *t*-test used for analysis can accommodate unequal group sizes.

Respondents were first provided with a consent form detailing the study's objectives, procedures, and participant rights, including an assent section to confirm their willingness to participate. Following this, a pre-test was administered to assess their baseline knowledge. The pre-test consisted of a multiple-choice exam evaluating students' understanding of multiplication concepts, problem analysis, and problem-solving skills. The questions were carefully crafted to focus on multiplying 2- to 3-digit numbers, ensuring the content was appropriate for their grade level.

During the implementation phase, the control group was taught multiplication using traditional methods, such as rote memorization, repeated practice, and direct instruction, to establish a baseline of student performance. Conversely, the experimental group was taught using the lattice method, an innovative approach to enhance students' understanding and performance in multiplication. Detailed lesson plans and teaching materials specific to the lattice method were used to ensure consistency and effectiveness in instruction.

After the instructional period, both groups were administered a post-test. The same set of questions was used in the post-test to measure the effectiveness of the intervention. This consistency allowed for a direct comparison of the results, making it possible to assess student performance improvements accurately. Using identical questions in both tests ensured that any observed changes could be attributed to the intervention rather than variations in test content. The data collected from the pre-tests and post-tests were then compiled and analyzed by a research statistician to determine the statistical significance of the results, providing a thorough assessment of the study's findings.

Data analysis

Content validity. The content validity of the tool was determined using Aiken's V coefficient. This statistical measure reflects the degree of consensus among experts regarding the relevance of each item to the learning objectives, its necessity, and the overall quality of the items included in the tool. In this study, the calculated Aiken's V coefficient was 0.95, indicating a strong validity level. This high value suggests that the tool was well-aligned with the intended learning objectives and was highly relevant for assessing the participants' knowledge and skills, as supported by the work of Sireci and Bond (2014).

Reliability. The tool's reliability was assessed using Cronbach's alpha, a widely used measure of internal consistency. The tool achieved a Cronbach's alpha value of 0.85, demonstrating high reliability. This value indicates that the tool consistently measured what it was intended to measure across different items and participants. The high-reliability score suggests that the tool was stable and produced dependable results, essential for ensuring that the data collected accurately reflects the participants' learning achievements, as noted by Ahdika (2017).

Mean. The mean was calculated to address the first and third statements of the problem. This statistical measure provided an average score that was then transmuted to the Department of Education (DepEd) grading system for interpretation. The DepEd grading scale, outlined in Table 1, was used to interpret the mean scores. The grading scale classified the scores as Outstanding (90-100), Very Satisfactory (85-89), Satisfactory (80-84), Fairly

Satisfactory (75-79), and Did Not Meet Expectations (Below 75). This conversion allowed for a standardized interpretation of the results, making assessing the participants' performance easier relative to established benchmarks.

Table 1. The DepEd grading scale.

Grading scale	Interpretation
90 - 100	Outstanding
85 - 89	Very satisfactory
80 - 84	Satisfactory
75 - 79	Fairly satisfactory
Below 75	Did not meet expectations

T-test of Independent Samples. The unequal sample is not a problem because the statistical tool is an independent *t*-test, not a paired sample *t*-test. Control and experimental groups need not be equal because they are independent groups and are warranted under this study. This was utilized to analyze whether significant differences existed between the pre-test and post-test scores of the control and experimental groups. This statistical test was crucial in addressing the second and fourth statements of the problem. By comparing the means of the two groups before and after the intervention, the *t*-test determined whether the instructional methods statistically impacted the participants' performance. The interpretation of the *t*-test results was based on the *p*-value; if the *p*-value was less than 0.05, it indicated a significant difference between the groups, suggesting that the intervention had a notable effect. If the *p*-value was 0.05 or greater, it suggested no significant difference, indicating that the instructional methods did not substantially impact the outcomes.

RESULTS

This study compared students' multiplication skills in an experimental group (n = 21) with a control group (n = 22) before adopting the lattice and long multiplication methods. Both groups received a pre-test to evaluate their baseline multiplication skills, specifically their ability to multiply up to 3-digit numbers by up to 2-digit numbers. The statistical analysis of the pre-test scores is summarized in Table 2. The data showed that neither the control nor the experimental group met the expectations set by the K to 12 grading system. The control group had achieved a mean score of 9.41, corresponding to a grade percentage of 65.68, while the experimental group had attained a mean score of 10.33, translating to a grade percentage of 67.22. Both percentages fell below the proficiency threshold.

Table 2. Average of pre-test multiplication score achievement between the control group and experimental group.

Group	Total scores	Standard deviation	Mean	Grade percentage	Remarks
Control	30	3.75	9.41	65.68	Did not meet expectations
Experimental	30	4.93	10.33	67.22	Did not meet expectations

A significant difference was analyzed between the mean pre-test scores of the control and experimental groups. The statistical analysis of these scores is summarized in Table 3. This indicated that the mean pre-test score for the control group was 9.41 with a standard deviation of 3.75, while the experimental group had a mean score of 10.33 with a standard deviation of 4.93. The high standard deviation of the experimental group indicates that the scores in this group are more diverse than those in the control group. The *t*-value was -0.689, and the *p*-value was 0.495, showing no significant difference between the two groups' pre-test scores. This lack of significant difference suggests that both groups started at a similar level of multiplication proficiency before introducing the respective teaching methods. The similar baseline performance indicates that any differences observed in the post-test results can be more confidently attributed to the intervention rather than pre-existing disparities in ability. The data also reinforce the notion that initial multiplication skills among students were generally low, highlighting the challenges students faced in mastering multiplication, irrespective of the teaching method initially employed.

Table 3. Mean comparison between pre-test scores of the control group and experimental group.

Group	Mean	Standard deviation	*t*-value	*p*-value	Interpretation
Control Experimental	9.41 10.33	3.75	-0.689	0.495	Pre-test scores between the two groups do not differ significantly.

The average post-test multiplication test scores for the control and experimental groups were examined, and the results were summarized in Table 5. The control group had a mean post-test score of 14.82 with a standard deviation of 6.86, resulting in a grade percentage of 74.70, which did not meet expectations. In contrast, the experimental group had a mean post-test score of 20.86 with a standard deviation of 4.55, achieving a grade percentage of 84.77, classified as very satisfactory. This significant difference in post-test scores suggests that the experimental group, which was taught using the lattice method, outperformed the control group, indicating the effectiveness of the lattice method in enhancing students' multiplication skills. The results highlight the potential of alternative teaching methods to improve learning outcomes, especially in areas where students traditionally struggle.

Table 4. Average post-test multiplication score achievement between the control group and experimental group.

Group	Total scores	Standard deviation	Mean	Grade percentage	Remarks
Control	30	6.86	14.82	74.70	Did not meet expectations
Experimental	30	4.55	20.86	84.77	Very satisfactory

The significant difference in post-test scores between the control and experimental groups was analyzed, with results summarized in Table 6. It revealed a substantial difference in post-test mean scores between the control and experimental groups. The high standard deviation of 6.86 in the control group suggests a more significant variability in the post-test multiplication scores compared to the experimental group, which has a standard deviation of 4.55. This means that the scores in the control group are more dispersed around the mean, indicating that students in this group had a more comprehensive range of performance levels. This suggests that students in the control group had very different levels of achievement on the post-test, reflecting inconsistency in their performance. Also, even though the intervention in the experimental group might have been successful on average, the high standard deviation indicates that not all students benefitted equally. Some students might have excelled, while others may have experienced the same improvement.

On the other hand, the *t*-value of -3.383 and a *p*-value of 0.002 indicated a statistically significant difference between the two groups. This significant difference suggests that the experimental group, which received instruction using the lattice method, demonstrated markedly improved multiplication skills compared to the control group, which followed traditional methods. The results confirm the effectiveness of the lattice method as a more successful teaching strategy in enhancing students' multiplication performance.

Table 5. Mean comparison between post-test scores of the control group and experimental group.

Group	Mean	Standard deviation	*t*-value	*p*-value	Interpretation
Control Experimental	14.82 120.86	6.86 4.55	-3.383	0.002	Pre-test scores between the two groups differ significantly.

DISCUSSIONS

The pre-test results indicate that the control and experimental groups struggled with multiplication, as neither met the expectations of the K to 12 grading system. These findings consistently indicate that students commonly face challenges in mastering multiplication, mainly when it involves more complex multi-digit numbers (Organisation for Economic Co-operation and Development, 2023; Ines, 2023; Balancio, 2023; Santos, 2023). Studies by Sullivan et al. (2015), Mulwa (2015), and Yayuk et al. (2020) similarly highlight the widespread difficulty students experience with multiplication, often due to cognitive and emotional factors.

The lack of a significant difference in pre-test scores between the control and experimental groups suggests that both groups began the study with similar levels of multiplication proficiency. This finding underscores any observed differences in the post-test results, likely due to the instructional methods rather than pre-existing differences in ability. The similarity in baseline performance aligns with the literature, indicating that challenges in multiplication are prevalent across different student populations (Sullivan et al., 2015; Mulwa, 2015; Yayuk et al., 2020).

The post-test results show a significant improvement in the performance of the experimental group, which used the lattice method, compared to the control group, which relied on traditional long multiplication methods. The lattice method's effectiveness in simplifying the multiplication process and reducing cognitive load aligns with findings from Rebekawati (2017), Javornik and Lipovec (2020), and Leighton (2020). These studies suggest that alternative teaching strategies emphasizing visual aids and step-by-step processes can significantly enhance students' understanding and performance in complex mathematical tasks.

The significant difference in post-test scores between the control and experimental groups supports the view that innovative teaching methods, like the lattice method, can overcome the limitations of traditional approaches. This result is consistent with Bruner's constructivist theory, which emphasizes the importance of scaffolding, visual representation, and building new knowledge on existing foundations (Ozdem-Yilmaz and Bilican, 2020; Rannikmäe et al., 2020; Donovan, 2021). The success of the lattice method in this study illustrates how breaking down complex tasks into manageable steps can improve student outcomes and align with best practices in math education.

The implications for math education are significant. The findings suggest that more than traditional methods may be required to improve students' multiplication skills, and alternative approaches like the lattice method should be considered to enhance students' learning experiences. By reducing cognitive load and simplifying complex procedures, the lattice method can help students achieve better results and gain confidence in their mathematical abilities, contributing to a more positive learning environment and improved academic performance (Boaler, 2015b;

Obongen et al., 2020; Banaszak, 2022). Thus, while a limited sample size means the findings should be interpreted cautiously, they can still offer valuable insights that may inform DepEd policies. The key is to frame the results as preliminary evidence that can guide further exploration, professional development, curriculum adjustments, and resource allocation while advocating for additional research to support broader implementation.

CONCLUSION

The study's results underscore the need for innovative approaches in Math Education, particularly for improving students' multiplication performance. Traditional teaching methods often fail to effectively address students' struggles with multiplication due to their complexity and the cognitive load involved, especially with regrouping. The lattice method has proven to simplify these steps, reducing cognitive strain and making the process more manageable, thus enhancing students' learning experiences and outcomes.

Given that the pre-test results did not meet K-12 grading standards, there should be a renewed focus on teaching the multiplication of 3-digit numbers by 2-digit numbers, potentially through additional support or remediation classes. Since there was no significant difference between the experimental and control groups' pre-test scores, any modifications to the teaching strategy should be applied uniformly across all Grade 4 sections to ensure equitable learning opportunities. The control group's post-test results, which still did not meet K-12 standards, suggest that traditional methods may need to be revised or supplemented with more innovative approaches. Additionally, the Department of Education (DepEd) may consider incorporating the lattice method into professional development sessions, such as Learning Action Cell (LAC) sessions, to equip teachers with practical strategies for teaching multi-digit multiplication.

ACKNOWLEDGEMENT

This article would not have been possible without the active participation and support of the respondents and educators at Cateel Central Elementary School—special thanks to the two Grade 4 sections for their engagement in demonstrations and exams. Our deepest gratitude goes to Principal Mr. Constantino R. Bagumba and advisers Mrs. Ingrid D. Tranquilan and Mrs. Mirasol Joy F. Vallejo for their approval and support.

REFERENCES

Arguel, A., Lockyer, L., Lipp, O. V., Lodge, J. M., and Kennedy, G. (2017). Inside out: Detecting learners' confusion to improve interactive digital learning environments. *Journal of Educational Computing Research*, 55(4), 526–551. https://doi.org/10.1177/0735633117703550

Baker, A. T., and Cuevas, J. (2018). The importance of automaticity development in mathematics. *Georgia Educational Researcher*, 14(2), 13–23. https://doi.org/10.20429/ger.2018.140202

Balancio, J. (2023, December 5). No improvement in learning? 2022 PISA results show Filipino learners still lagging behind in math, reading, science. ABS-CBN News.

Banaszak, B. (2022). Lattice method of multiplication: Overview and examples. Study.com.

Boaler, J. (2015a). Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching. John Wiley & Sons.

Boaler, J. (2015b, January 28). Fluency without fear: Research evidence on the best ways to learn math facts. Schudio.

Cardino, J. M., and Ortega-Dela Cruz, R. A. (2020). Understanding of learning styles and teaching strategies towards improving the teaching and learning of mathematics. *LUMAT International Journal on Math Science and Technology Education*, 8(1). https://doi.org/10.31129/lumat.v8i1.682

Chevalier, J. M., and Buckles, D. J. (2019). Participatory action research: Theory and methods for engaged inquiry. Routledge.

Del Carmen Chamorro, M. (2021). Can we explain students' failure in learning multiplication? *In Teaching multiplication with lesson study* (pp. 265–289). Springer International Publishing. https://doi.org/10.1007/978-3-030-56983-5_12

Donovan, M. M. O. (2021, July 19). An overview of Bruner and Piaget—Cognitive constructivists. Pure.

Dotan, D., and Zviran-Ginat, S. (2022). Elementary math in elementary school: The effect of interference on learning the multiplication table. Cognitive Research: *Principles and Implications*, 7(1). https://doi.org/10.1186/s41235-022-00377-5

Gardiner, T. (2016). Teaching mathematics at secondary level. Open Book Publishers.

Goos, M., Vale, C., Stillman, G., Makar, K., Herbert, S., and Geiger, V. (2020). Teaching secondary school mathematics: Research and practice for the 21st century. Routledge.

Ines, J. (2023). PH still among lowest in math, science, reading in global student assessment. Rappler.

International Commission on Mathematical Instruction (ICMI). (2023). The role of mathematics in the overall curriculum. Math Union.

Javornik, I., and Lipovec, A. (2020). Do alternative algorithms for two-digit multiplication really help students to be more efficient? *Academia*. https://doi.org/10.13140/RG.2.2.23820.90241

Larsson, K. (2016). Students' understandings of multiplication. Diva Portal.

Leighton, L. (2020). What is the lattice method of multiplication? ResearchGate. https://doi.org/10.13140 RG.2.2.22930.94409

Lewis, C. (2016). How does lesson study improve mathematics instruction? *ZDM*: The International Journal on Mathematics Education, 48(4), 571–580. https://doi.org/10.1007/s11858-016-0794-7

Lopez, L. A. (2020). Improving teachers' conceptual knowledge of fractions through online subject-specific professional development. *Journal of Mathematics Teacher Education*. https://doi.org/10.1007/s10857-020-09514-0

Milton, J. H., Flores, M. M., Moore, A. J., Taylor, J. J., and Burton, M. E. (2019). Using the concrete–representational–abstract sequence to teach conceptual understanding of basic multiplication and division. Learning Disability Quarterly: *Journal of the Division for Children with Learning Disabilities*, 42(1), 32–45. https://doi.org/10.1177/0731948718801276

Moussa-Inaty, J., Causapin, M., and Groombridge, T. (2020). Does language really matter when solving mathematical word problems in a second language? A cognitive load perspective. *Educational Studies,* 46(1), 18–38. https://doi.org/10.1080/03055698.2019.1670377

Mulwa, E. C. (2015). Difficulties encountered by students in the learning and usage of mathematical terminology: A critical literature review. *Journal of Education and Practice*, 6(13), 27-37.

Newton, N., Record, A. E., and Mello, A. J. (2024). Fluency doesn't just happen in multiplication and division: Strategies and models for teaching the basic facts. Taylor & Francis.

Obongen, A. V., Allauigan, L. M., Carpo, D. N., and Pablo, M. N. (2020). The effect of introducing the box/lattice method to the competency of Grade 9 students of Holy Spirit National High School in multiplying polynomials. *European Journal of Humanities and Educational Advancements,* 1(4), 42-47.

Organisation for Economic Co-operation and Development. (2023). PISA 2022 results: Factsheets Philippines.

Ozdem-Yilmaz, Y., and Bilican, K. (2020). Discovery Learning—Jerome Bruner. Science education in theory and practice: An introductory guide to learning theory, 177-190.

Özenç, M., Dursun, H., and Şahin, S. (2020). The effect of activities developed with web 2.0 tools based on the 5E learning cycle model on the multiplication achievement of 4th graders. *Participatory Educational Research*, 7(3), 105–123. https://doi.org/10.17275/per.20.28.7.3

Qetrani, S., Ouailal, S., and Achtaich, N. (2021). Enhancing students' conceptual and procedural knowledge using a new teaching approach of linear equations based on the equivalence concept. *Eurasia Journal of Mathematics Science and Technology Education,* 17(7),em1978. https://doi.org/10.29333/ejmste/12408

Rannikmäe, M., Holbrook, J., and Soobard, R. (2020). Social constructivism—Jerome Bruner. In Springer texts in education (pp. 259–275). *Springer International Publishing*. https://doi.org/10.1007/978-3-030-23718-0_15

Rebekawati, A. (2017, January). The effect of using the lattice method on multiplication achievement among elementary students. ResearchGate.

Richey, J. E., Andres-Bray, J. M. L., Mogessie, M., Scruggs, R., Andres, J. M. A. L., Star, J. R., Baker, R. S., and McLaren, B. M. (2019). More confusion and frustration, better learning: The impact of erroneous examples. *Computers & Education*, 139, 173–190. https://doi.org/10.1016/j.compedu.2019.05.006

Rineck, L. M. (2020). A holistic approach to developmental mathematics. University of Wisconsin Milwaukee.

Rizzo, A. (2023). The golden ratio theorem: A framework for interchangeability and self-similarity in complex systems. *Advances in Pure Mathematics*, 13(09), 559–596. https://doi.org/10.4236apm.2023.139032

Santos, C. (2023). Filipino students still lagging behind in science, reading, math - PISA. RepublicAsia Media, Inc.

Sarkingobir, Y., Egbebi, L. F., and Awofala, A. O. A. (2023). Bibliometric analysis of the thinking styles in math and its implications on science learning. *International Journal of Essential Competencies in Education*, 2(1), 75–87. https://doi.org/10.28933/ijec-2023-01-0701

Sullivan, P., Askew, M., Cheeseman, J., Clarke, D., Mornane, A., Roche, A., and Walker, N. (2015). Supporting teachers in structuring mathematics lessons involving challenging tasks. *Journal of Mathematics Teacher Education*, 18(2), 123–140. https://doi.org/10.1007/s10857-014-9295-4

Thomas, L. (2020, July 31). Quasi-experimental design. Scribbr.

Van der Ven, S. H. G., Straatemeier, M., Jansen, B. R. J., Klinkenberg, S., and van der Maas, H. L. J. (2015). Learning multiplication: An integrated analysis of the multiplication ability of primary school children and the difficulty of single digit and multidigit multiplication problems. *Learning and Individual Differences*, 43, 48–62. https://doi.org/10.1016/j.lindif.2015.08.004

Yayuk, E., Purwanto, As'ari, A. R., and Subanji. (2020). Primary school students' creative thinking skills in mathematics problem solving. *European Journal of Educational Research*, 9(3), 1281–1295. https://doi.org/10.12973/eu-jer.9.3.1281